Image Reconstruction Method for an X-ray Telescope with an Angular Resolution Booster

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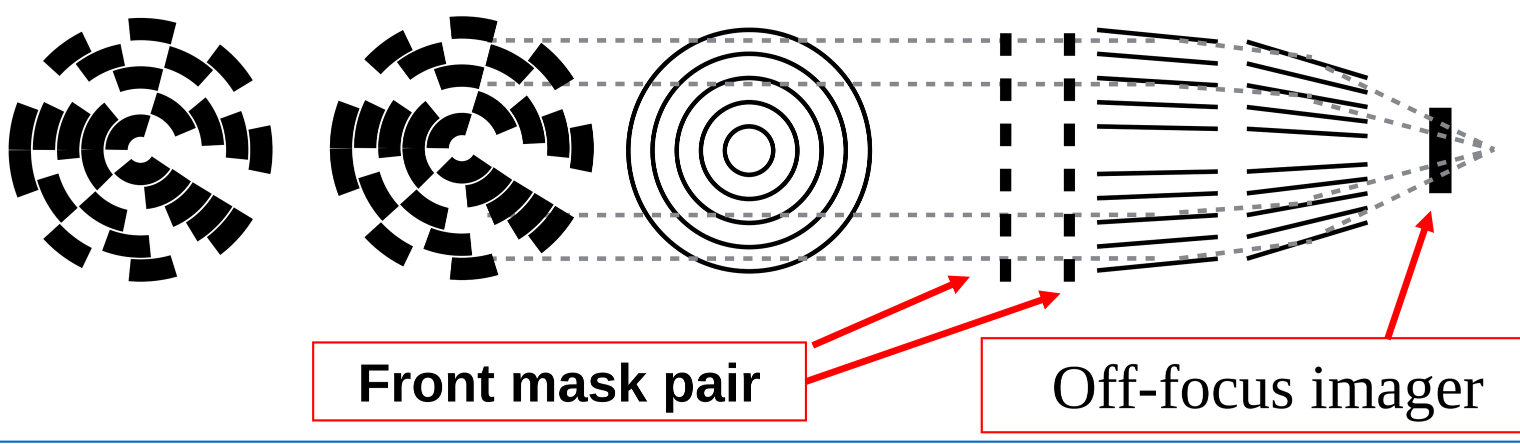
High angular resolution and large photon statistics are important for X-ray telescopes, but good instruments are expensive. Yoshitomo Maeda et al. [5] proposed a simple angular resolution booster for an X-ray telescope. It consists of double masks in front of an X-ray mirror and an imager at the off-focused plane. The image taken at the imager is a shadow pattern of the double masks, which contains the information of astronomical objects in high angular resolution. To get the sky image of astronomical objects, image reconstruction process must be applied. This idea is so simple that it can realize an X-ray telescope with large effective area in wide energy range and high angular resolution with low cost. Here, we propose an image reconstruction method for this system. Our method is based on the traditional Richardson-Lucy algorithm [7,4] with extension of two regularization terms of sparseness and smoothness. Such regularizations are desirable for astronomical imaging because astronomical objects have variety in shape from point sources, diffuse sources to mixtures of them. We use the EM algorithm [2] and the proximal gradient method [1] for the optimization. As a result, the image resolution can be improved from a few arcmin without the booster down to a few arcsec with the booster. The performance is demonstrated with simulated data: double point sources and diffused sources such as Cas A and Crab Nebula. Through the demonstration, our proposing system is shown to be feasible [6].

Angular Resolution Booster (arcmin → arcsec)



Even if angular resolution of an X-ray mirror is about a few arcmin, that of the reconstructed image can be a few arcsec, because the resolution is determined by the front mask.

Image taken at the off-focus imager

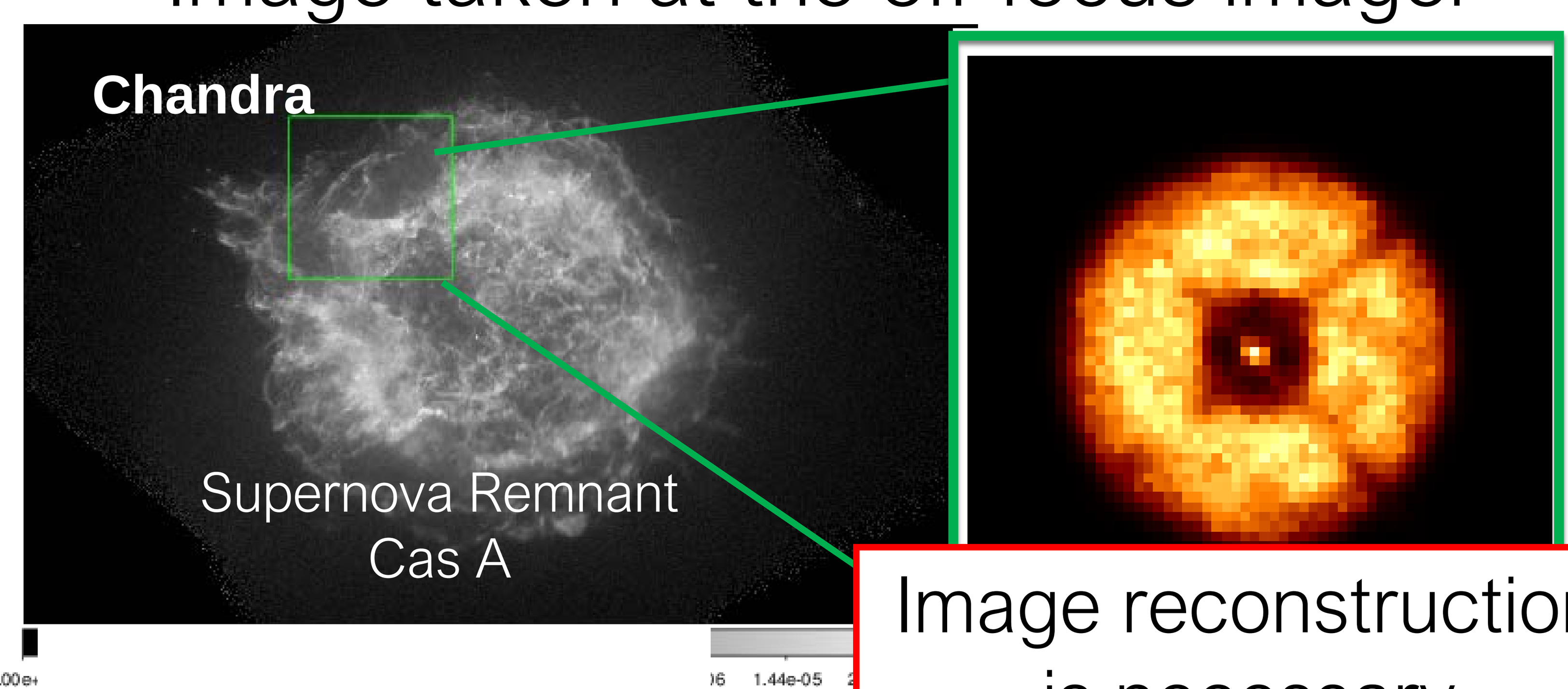


Image reconstruction is necessary

Mathematical Setup

The number of photons detected at the off-focus imager follows the Poisson distribution.

$$Y(v) \sim \text{Poisson}\left(\sum_u t(v,u)\lambda(u)\right)$$

- ✓ Y(v) : The number of photons at the pixel (v) of the off-focus imager
- ✓ T(v, u) : Response of the detector $\sum_v t(v,u) = 1$
- ✓ λ(u) : Image on the celestial sphere

Log likelihood: $L(\lambda) = \log \prod_v Y(v)$

Maximization of the Likelihood: Richardson-Lucy method

$$\lambda^{(r+1)} \leftarrow \sum_v \frac{t(v,u)\lambda(u)^{(r)}}{\sum_u t(v,u)\lambda(u)^{(r)}} Y(v)$$

It is the EM algorithm applied for the Poisson statistics.

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Sparseness (Dirichlet prior): EM algorithm

$$\log[\pi_\beta(\lambda)] = (\beta - 1)\sum_u \log\left[\frac{\lambda(u)}{\sum_u \lambda(u)}\right]$$

S. Ikeda et al. (2014) NIM A, 760, 46 [3]

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Smoothness: proximal gradient method + EM algorithm

$$V_\rho(\lambda) = \frac{1}{(\sum_u \lambda)^2}\sum_{(i,j)}\left[(\lambda_{i,j} - \lambda_{i+1,j})^2 + (\lambda_{i,j} - \lambda_{i,j+1})^2\right]$$

To minimize the difference between adjoining pixels

This work

Applicable for point sources and diffuse sources

Optimization Method

$$\lambda^* = \underset{\lambda(u)\geq 0}{\arg\max}\left[L(\lambda) + \log[\pi_\beta(\lambda)] - \mu V_\rho(\lambda)\right]$$

Log likelihood | Sparseness $\beta \in [0, 1]$ | Smoothness $\mu \in [0, \infty)$

1. EM algorithm

   M-step (r = 1,2, …)

$$\rho^{(r+1)} = \underset{\rho \in C}{\arg\min}\left[L_{\text{sub}} \equiv -\sum_u m_u^{(r)} \log \rho_u + (1-\beta)\sum_u \log \rho_u + \mu V(\rho)\right],$$

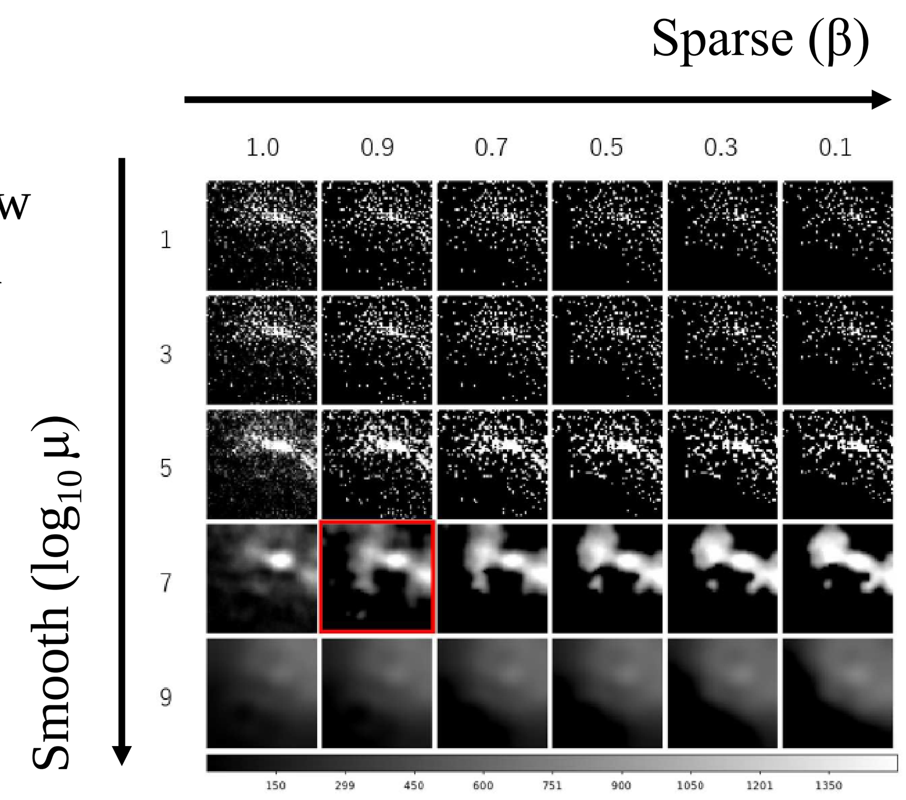
$$m_u^{(r)} = \sum_v Y(v) \frac{t(v,u)\rho_u^{(r)}}{\sum_u t(v,u)\rho_u^{(r)}}.$$

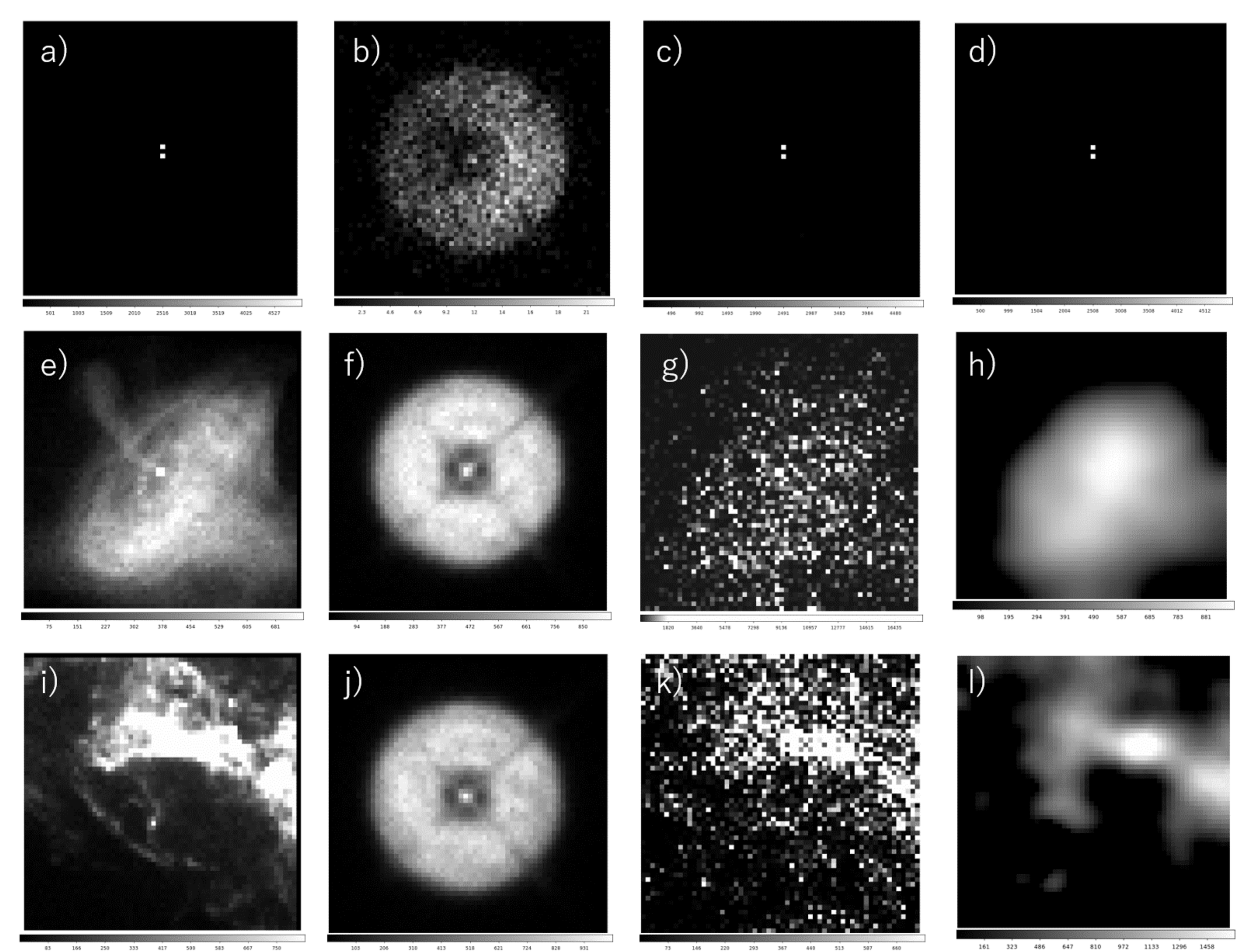
2. EM M-step is solved by the proximal gradient method.

Demonstration

The input images are shown in the figure below (a, e, i), and the simulated observed images on the off-focus imager are the panels (b, f, j), respectively. The reconstructed images by Richardson-Lucy method and our method are shown in 3rd and 4th columns.

Optimal hyper parameters (μ, β) is determined by the cross-validation (10 fold) as shown in the right figure (The best one is in red frame).





- ✓ Double stars with a separation of 4 arcsec are successfully separated.
- ✓ Our method can reconstruct diffused sources well, while those by Richardson-Lucy method are noisy.
- ✓ 1-2 order of magnitude improvement of angular resolution can be realized by using angular resolution booster.

References [1] A. Beck & M. Teboulle, SIAM J. Imaging Sci., 2, 183 (2000), [2] A. P. Dempster, N. M. Laird & D. B. Rubin, Journal of the Royal Statistical Society. Series B, 39, 1 (1977), [3] S. Ikeda, et al., NIM A, 760, 46 (2014), [4] L. B. Lucy, AJ, 79, 745 (1974), [5] Y. Maeda, et al., PASJ, submitted (2018), [6] M. Morii, S. Ikeda, & Y. Maeda, PASJ, submitted (2018), [7] W. H. Richardson, Journal of the Optical Society of America, 62, 55 (1972)

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